

X-ray Fabry-Pérot interferometers

or

interferometry with μeV -resolution

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$$\Delta E \Delta t = \hbar$$

$$\Delta E = 1 \mu\text{eV} \quad \Rightarrow \quad \Delta t = 658 \text{ ps}$$

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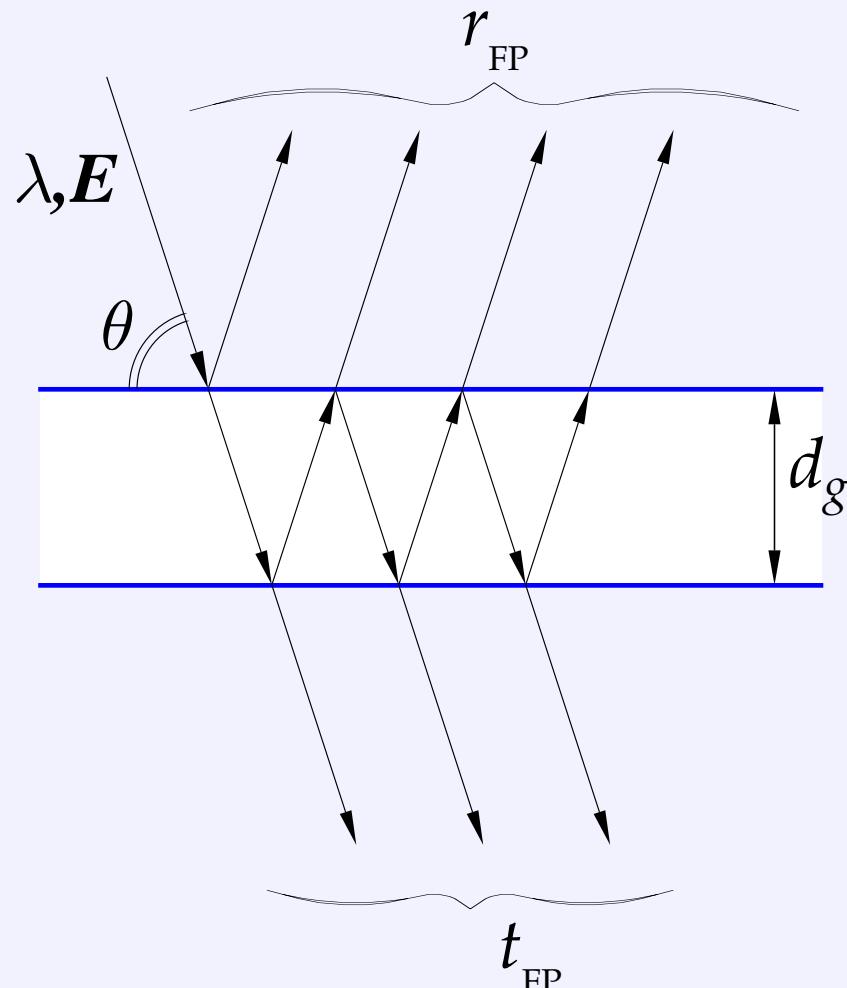
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Content

- From optical to x-ray Fabry-Pérot interferometers.
- How do x-ray Fabry-Pérot interferometers work?
- Technical challenges.
- Prototype x-ray Fabry-Pérot interferometer.
- Potential applications of x-ray Fabry-Pérot interferometers.

Optical Fabry-Pérot Interferometer (FPI)



1. Two parallel mirrors of high reflectivity R .
2. Multiple reflections
⇒ Superposition of many partial waves
⇒ Multiple-wave interference .
3. Despite high reflectivity of the mirrors, the system becomes transparent if the condition for the formation of the standing wave in the gap between the mirrors is fulfilled:

$n \lambda = 2d_g \sin \theta$

4. Parameters: d_g , θ , λ oder E .

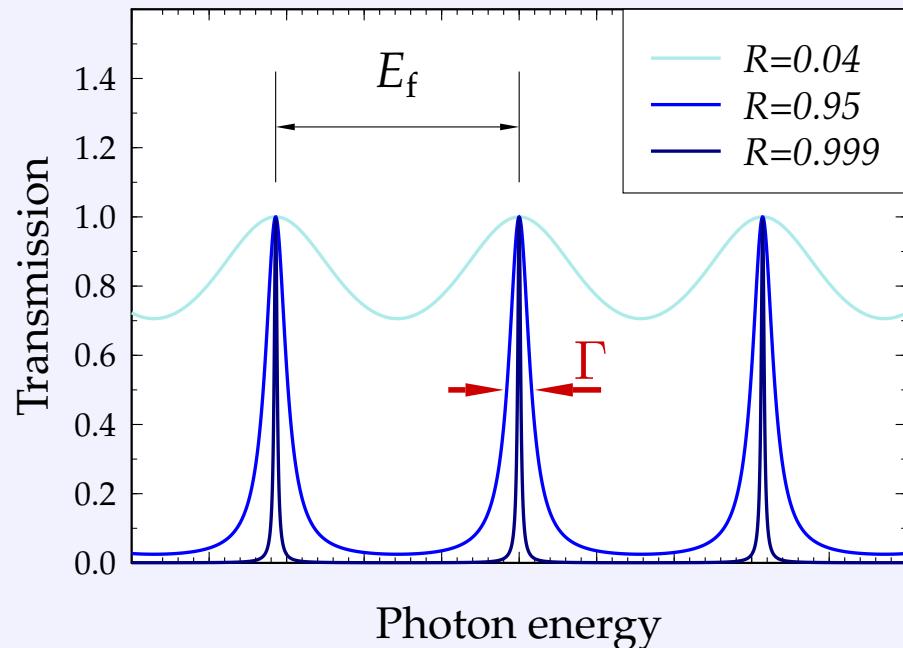
Energy dependence of transmission through a FPI

$$\theta = 90^\circ$$

$$E_f = \frac{hc}{2d_g} = \text{free spectral range}$$

$$\Gamma = \frac{E_f}{F}$$

$$F = \frac{\pi\sqrt{R}}{1-R} = \text{Finesse}$$



Example: for $R = 0.95$ and $d_g = 1 \text{ cm}$

$$F = 61.2 \quad E_f = 62 \mu eV$$

$$\Gamma = 1 \mu eV$$

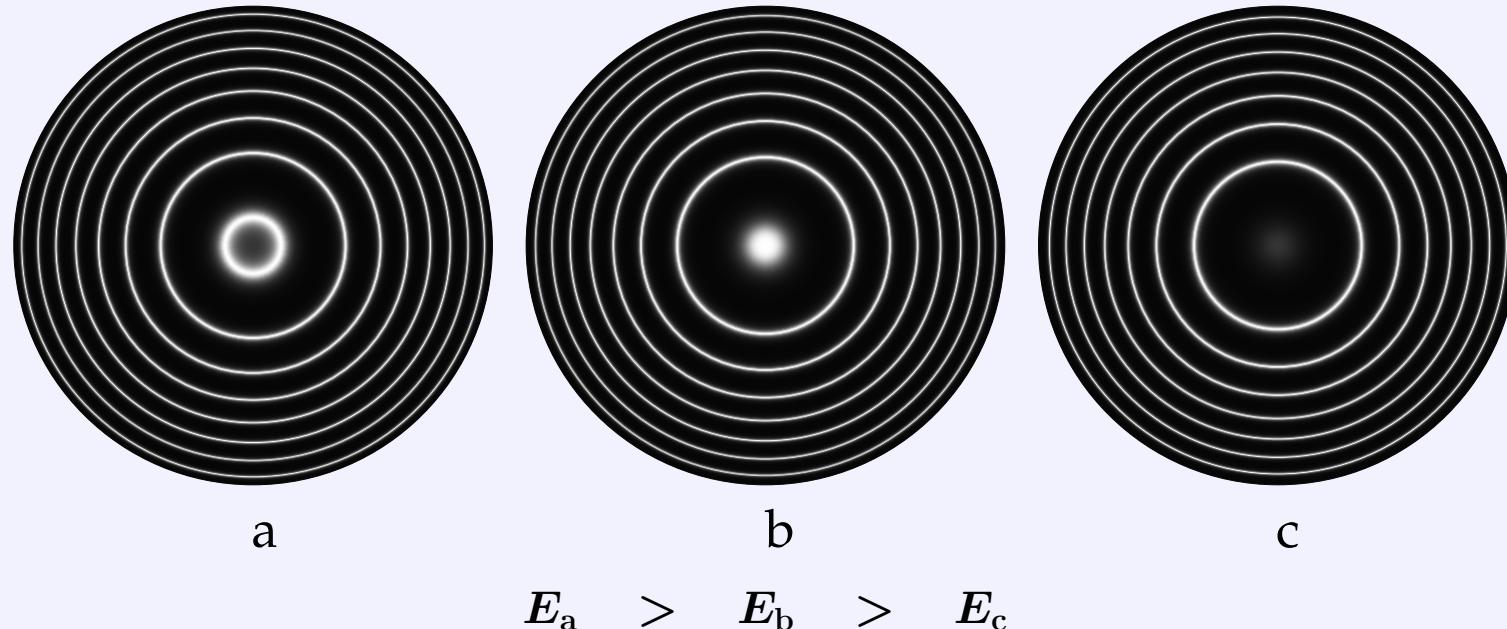
⇒ The results are independent of the photon energy!

⇒ i.e., applicable also in x-ray spectral region.

X-ray mirrors of very high reflectivity are required !!!.

Angular dependence of transmission through a FPI

Multiple-wave interference rings.



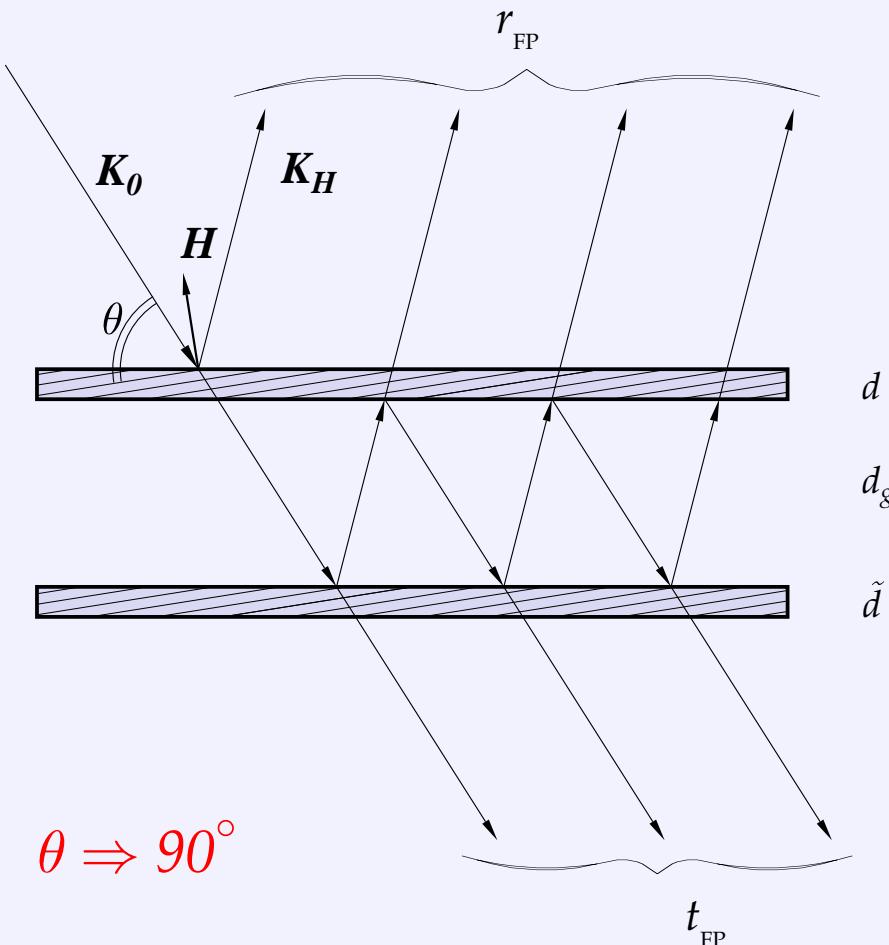
⇒ The largest angular acceptance is at normal incidence!

⇒ Normal incidence x-ray mirrors of high reflectivity are required !!!

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X-ray Fabry-Pérot interferometer



X-ray Fabry-Pérot Interferometer with two parallel single crystal plates as reflecting mirrors.

Fabry-Perot-type interference filter for x-rays

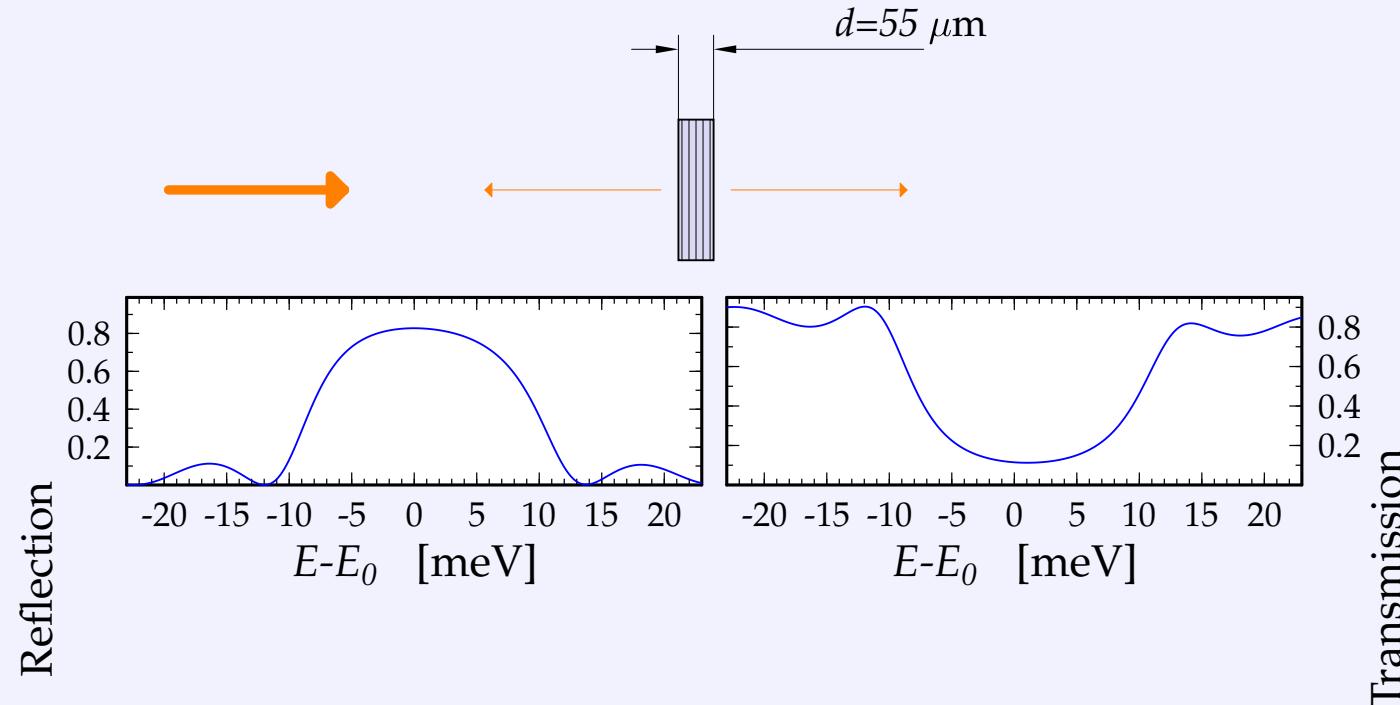
STEYERL and STEINHAUSER (1979).

The back reflection of the optical mirrors is proposed to replace by the Bragg reflection of crystal plates reflecting at normal incidence.

Further theoretical treatments:
CATICHA, CATICHA-ELLIS (1981)
SHVYD'KO, GERDAU (1999)
KOHN, SHVYD'KO, GERDAU (2000)
SHVYD'KO (2001).

X-ray Fabry-Pérot interferometer: How does it work (1)

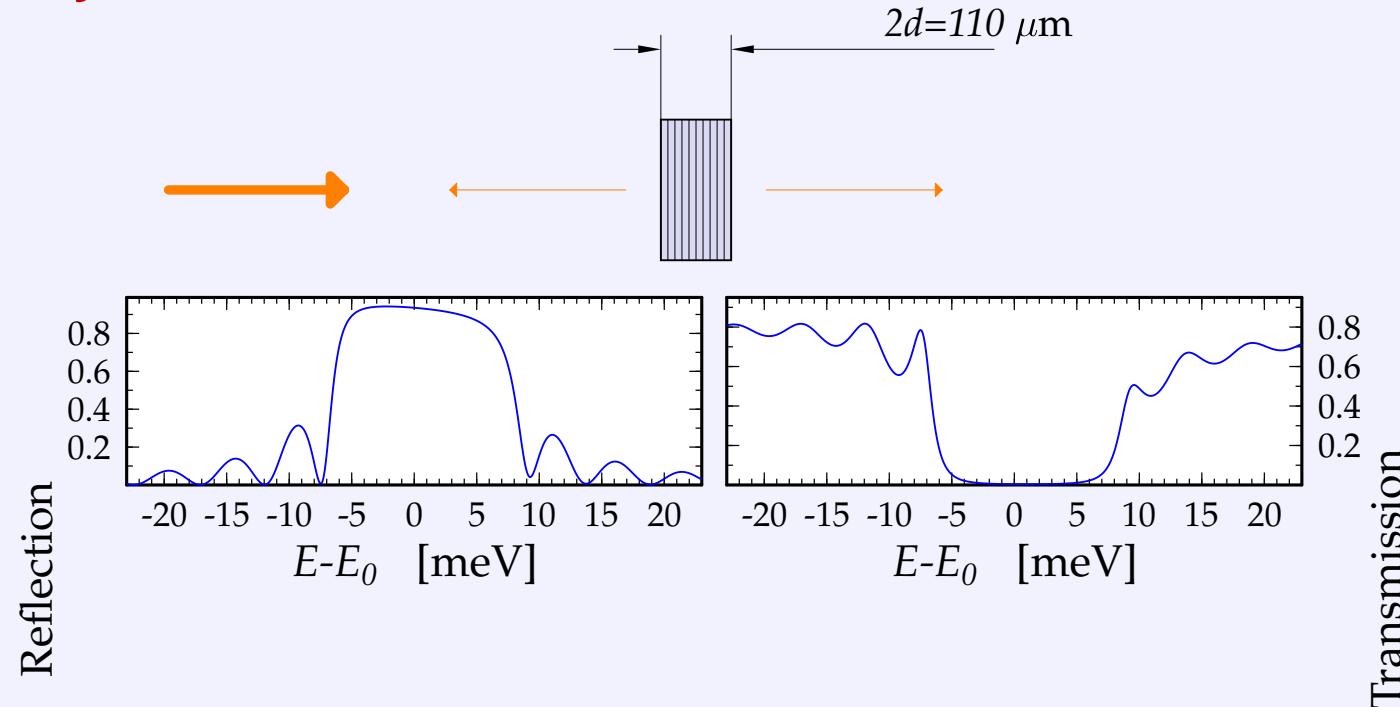
Transmission and reflection at normal incidence to the atomic planes:



$\text{Al}_2\text{O}_3(0\ 0\ 0\ 30)$, $E_0 = 14.315 \text{ keV}$

X-ray Fabry-Pérot interferometer: How does it work (2)

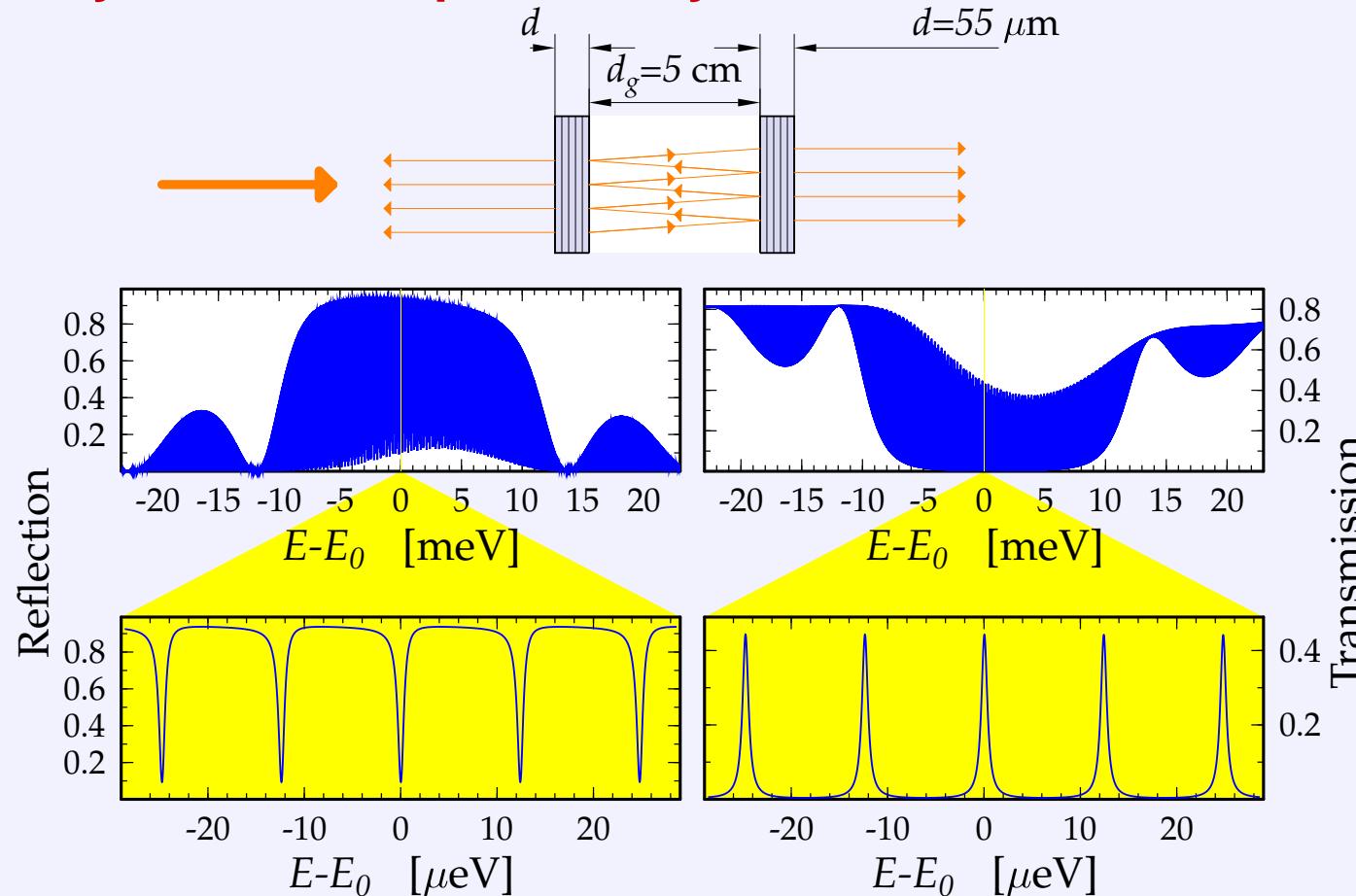
Crystal of a double thickness:



$\text{Al}_2\text{O}_3(0\ 0\ 0\ 30)$, $E_0 = 14.315 \text{ keV}$

X-ray Fabry-Pérot interferometer: How does it work (3)

A system of two parallel crystals:



$\text{Al}_2\text{O}_3(0\ 0\ 0\ 30)$, $E_0 = 14.315 \text{ keV}$

$$E_f = 12 \mu\text{eV}$$

$$\Gamma = 0.73 \mu\text{eV}$$

$$\Gamma/E_0 \simeq 5 \times 10^{-11}$$

$$\Delta\theta = 2\sqrt{\Gamma/E_0} \simeq 15 \mu\text{rad}$$

Content

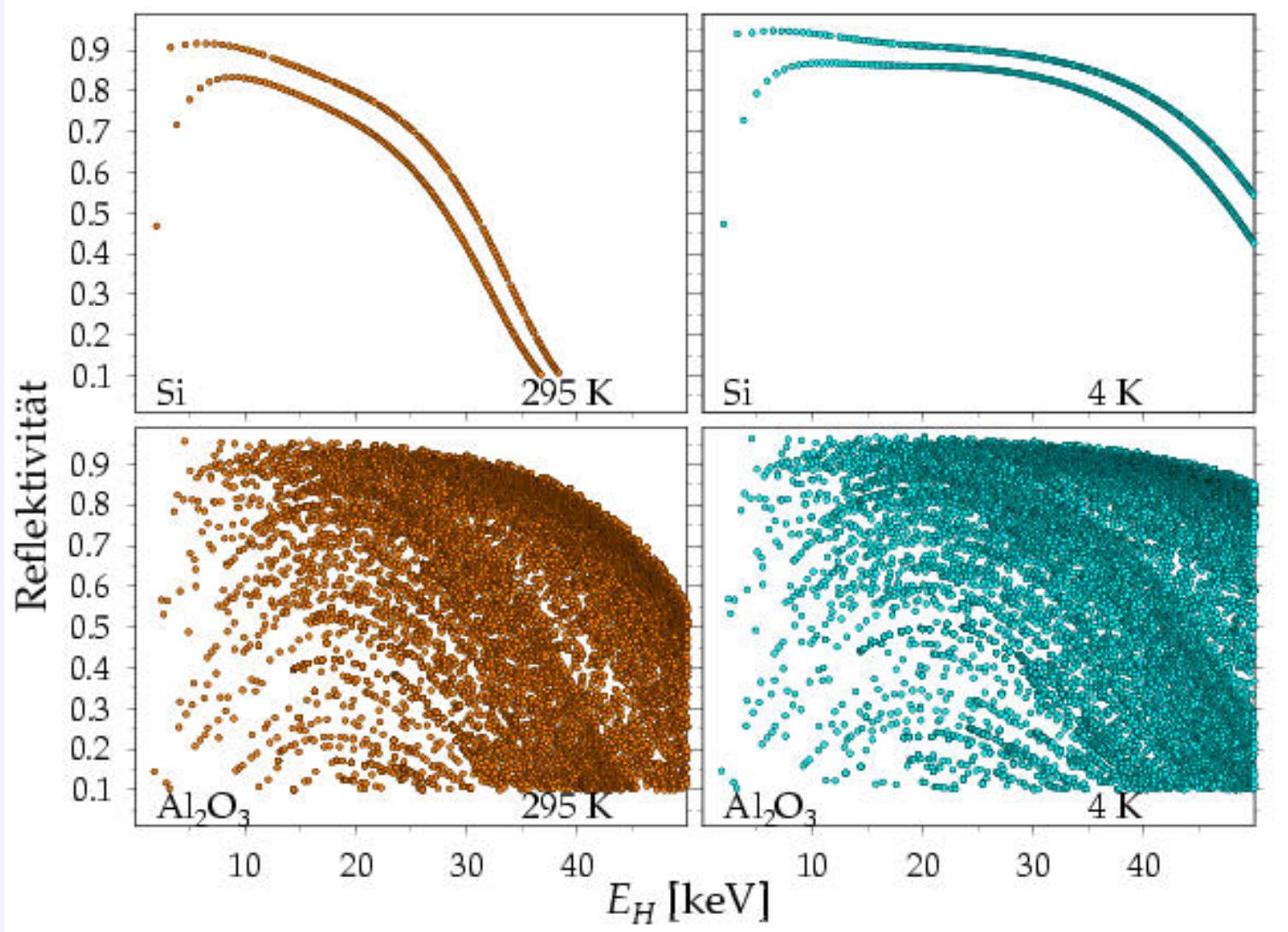
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Technical challenges of x-ray Fabry-Pérot interferometers

1. **Perfect** crystals as normal incidence Bragg-mirrors of high reflectivity. Best of all: Al₂O₃, C, Be, ... but not Si.

±

Reflectivity of Si und Al_2O_3 at normal incidence

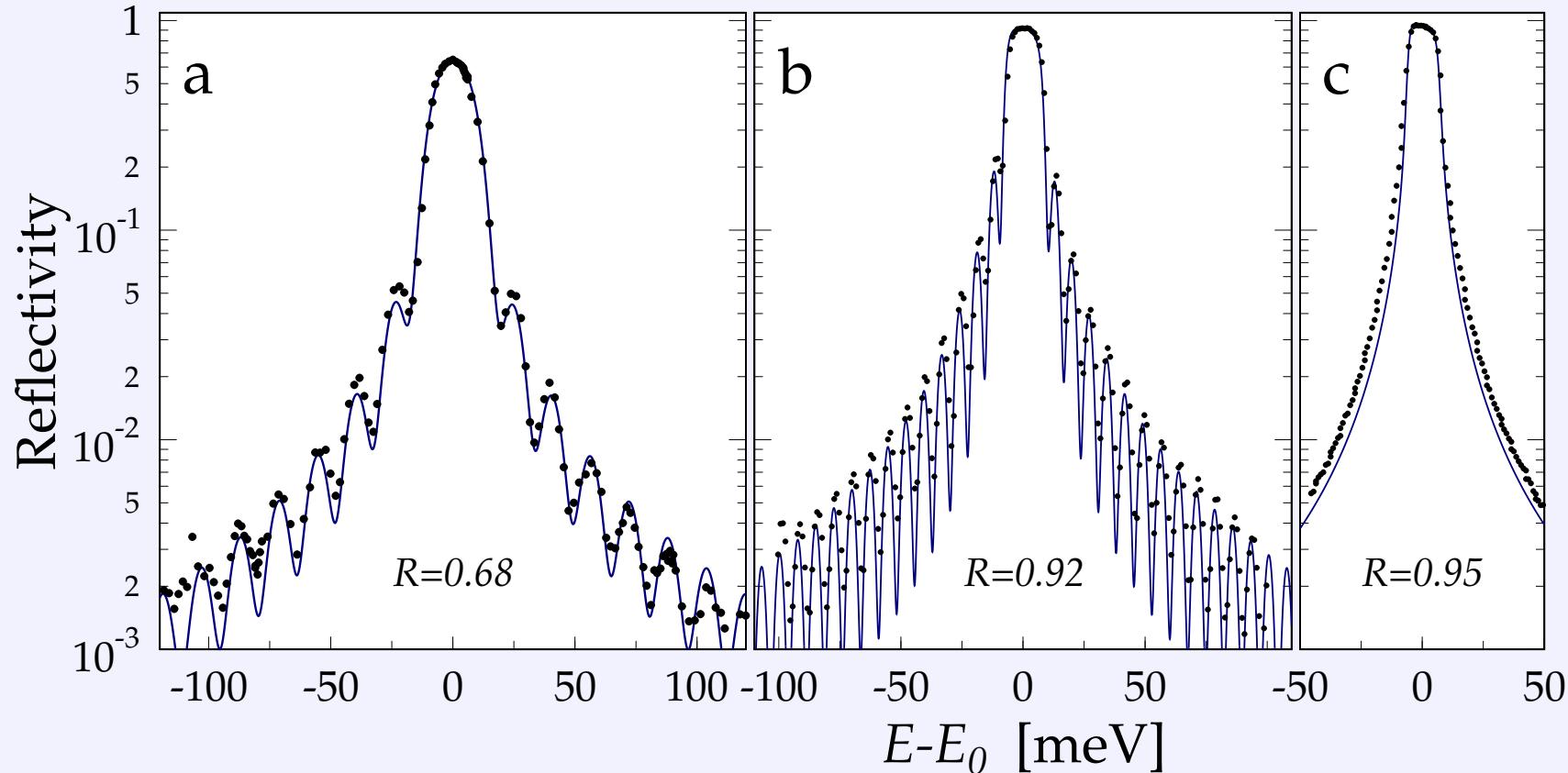


**Simulations
with the dynamical theory
for all allowed
Bragg reflections (2-beam
case, $d \rightarrow \infty$).**

Sapphire (Al_2O_3) normal incidence mirrors

Bragg reflectivity as function of the photon energy E for Al_2O_3 crystals of thickness d :

(a) $d = 39 \mu\text{m}$, (b) $d = 84 \mu\text{m}$, and, (c) $d = 2.5 \text{ mm}$.



Normal incidence at the (0 0 0 30) atomic planes in Al_2O_3 . $E_0 = 14.315 \text{ keV}$.
Solid lines are calculations with the dynamical theory.

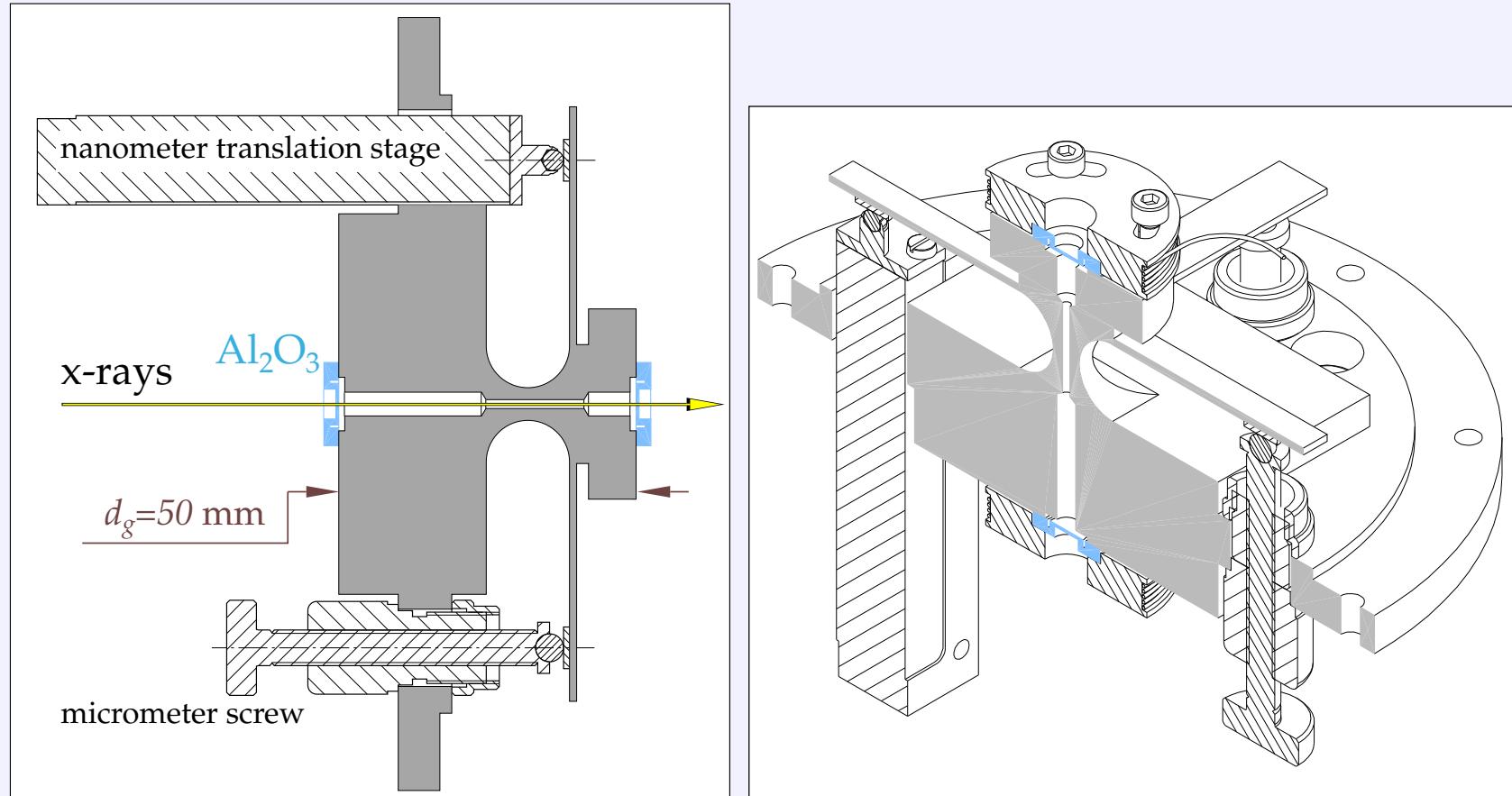
Technical challenges of x-ray Fabry-Pérot interferometers

1. **Perfect** crystals as normal incidence Bragg-mirrors of high reflectivity. Best of all: Al₂O₃, C, Be, ... but not Si.
±
2. **Nanoradian** - angle adjustment of the mirrors. +
3. **mK**-temperatur control. +
4. X-ray detectors with **ps**-time resolution. +
5. **0.01 nm**-precise linear positioning of the mirrors. -

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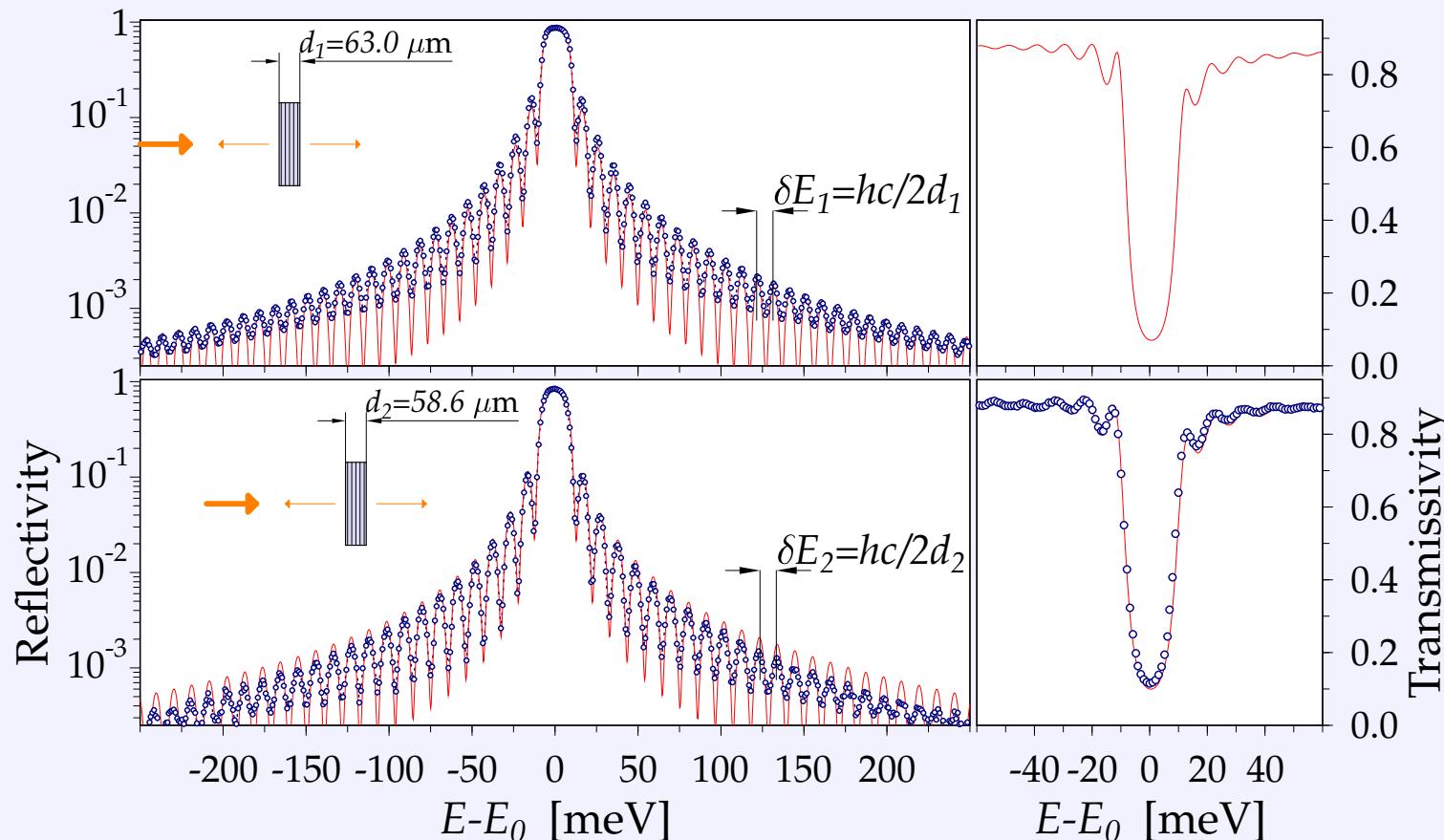
Design



The technical design of the x-ray resonator was aimed at allowing:
(i) parallel alignment of the mirrors with nanoradian accuracy;
(ii) temperature stability and homogeneity within 5 mK.

Interferometer mirrors

Energy dependence of the reflectivity (left) and transmissivity (right)

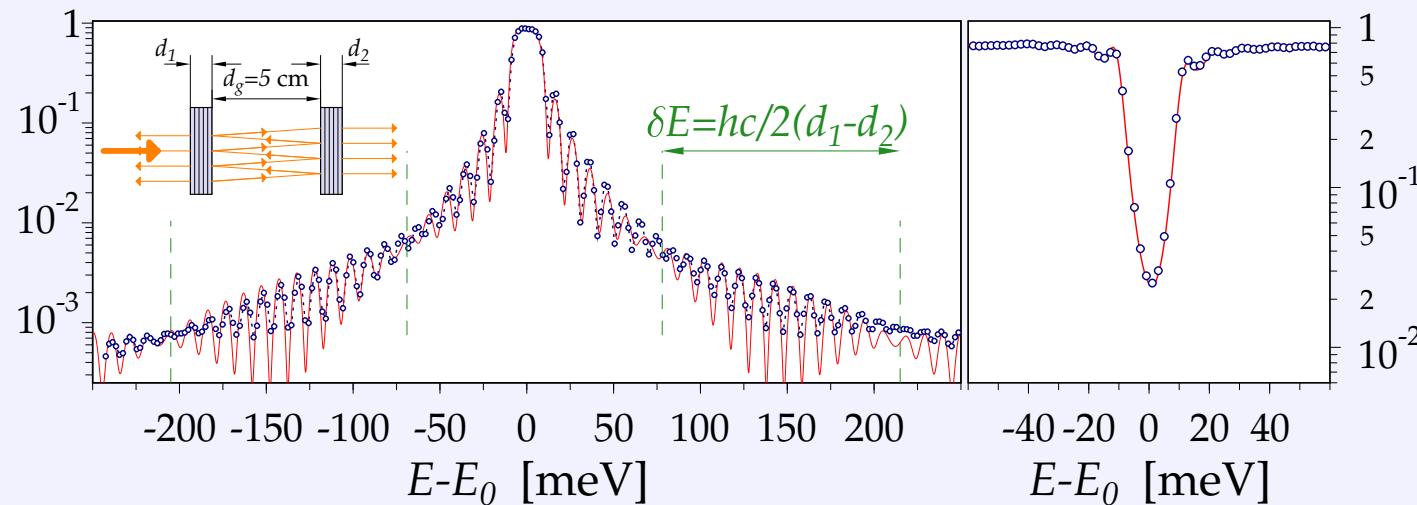


... of x-rays at normal incidence to the (0 0 0 30) atomic planes of Al_2O_3 single crystals
used as the interferometer mirrors. $E_0 = 14.315 \text{ keV}$.

With $R = 0.84(2)$ a Fabry-Pérot Interferometer should have a finesse of $F = 18(3)$.

Two-crystal interferometer

Energy dependence of the reflectivity (left) and transmissivity (right)



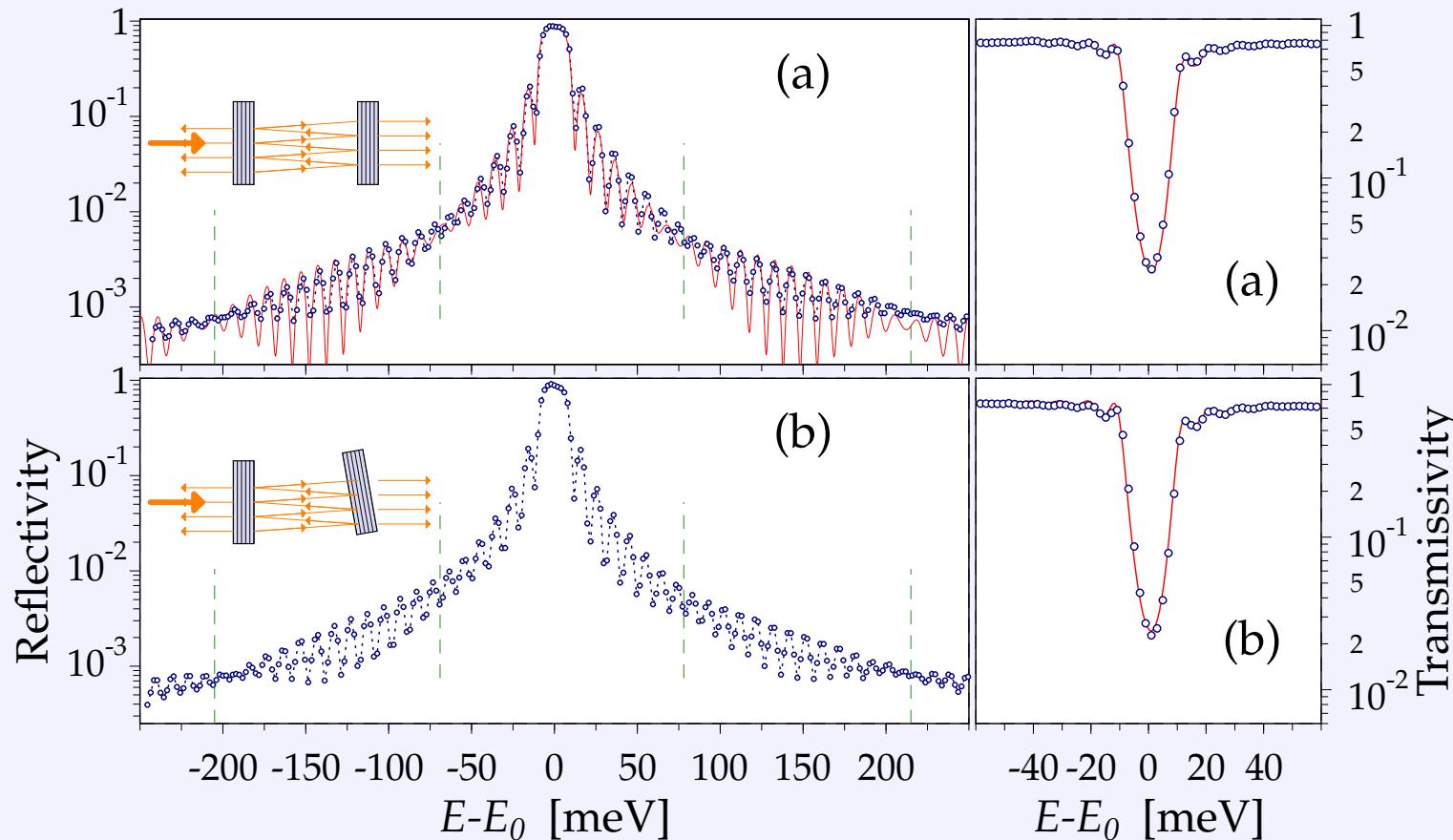
The atomic planes of the both crystals are adjusted parallel to better than $0.35 \mu\text{rad}$.

$$E_0 = 14.315 \text{ keV}$$

The solid lines are calculated by the theory of x-ray Fabry-Pérot interferometers.

Two-crystal interferometer

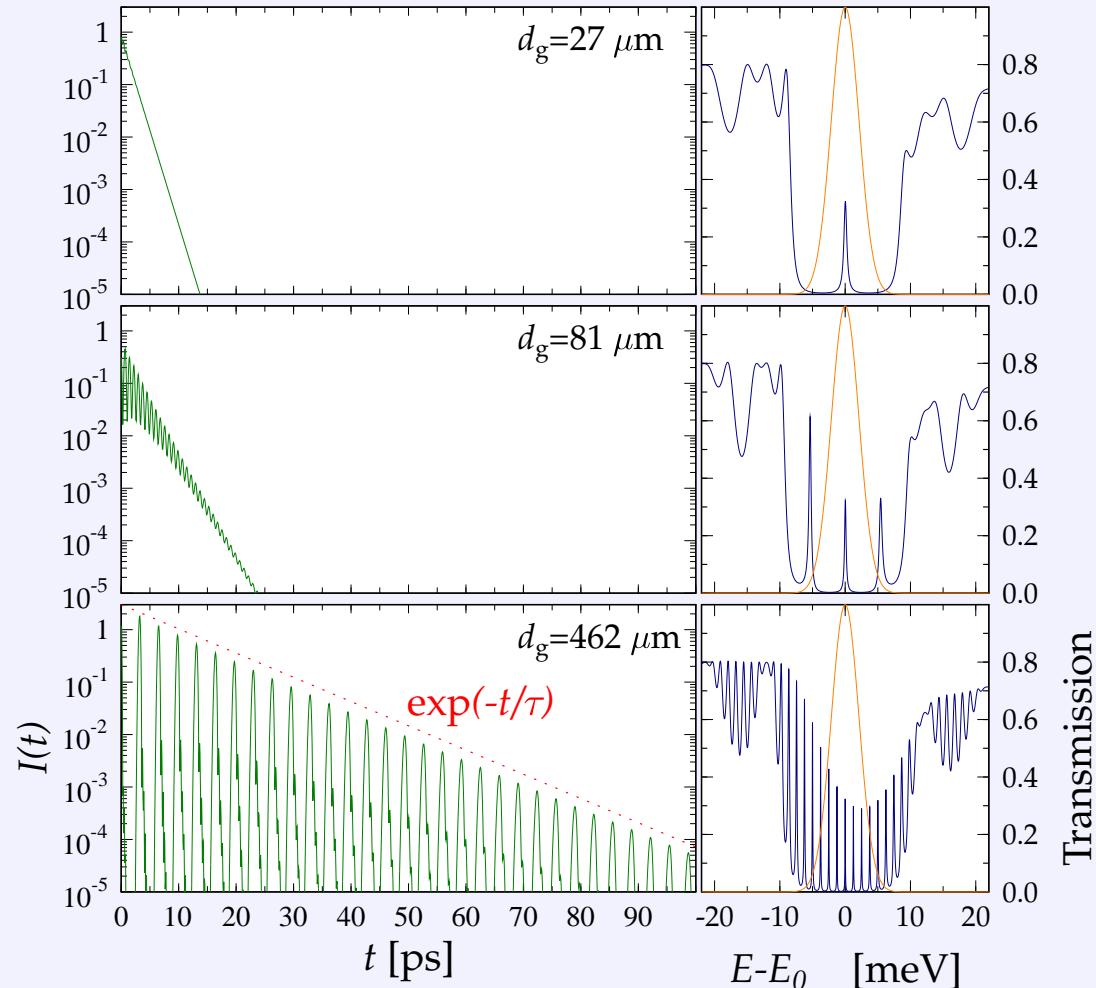
Energy dependence of the reflectivity (left) and transmissivity (right)



(a): The atomic planes of the both crystals are adjusted parallel to better than $0.35 \mu\text{rad}$.

(b): The mirror 2 is tilted by $3 \mu\text{rad}$ from the parallel state.

Time response of an x-ray Fabry-Pérot interferometer



How to observe μeV -sharp resonances?

With the help of the time response of the interferometer!
 E and t are complementary!

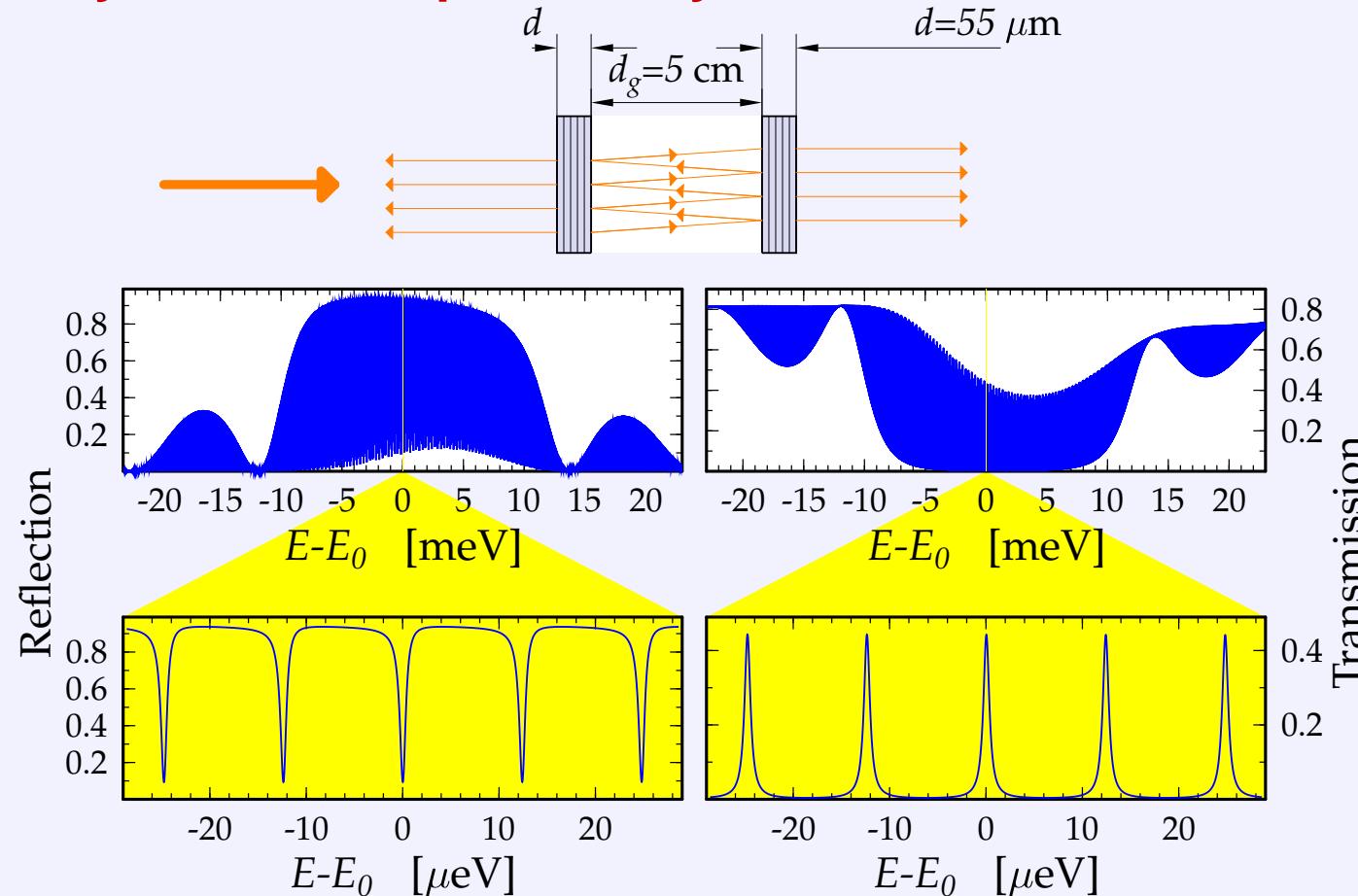
$$\tau = \frac{\hbar}{\Gamma} = \text{decay time}$$

$$t_f = \frac{\hbar}{E_f} = \text{quantum-beat period}$$

$$t_f = \frac{2d_g}{c} = \text{time-of-flight}$$

X-ray Fabry-Pérot interferometer: How does it work (3)

A system of two parallel crystals:



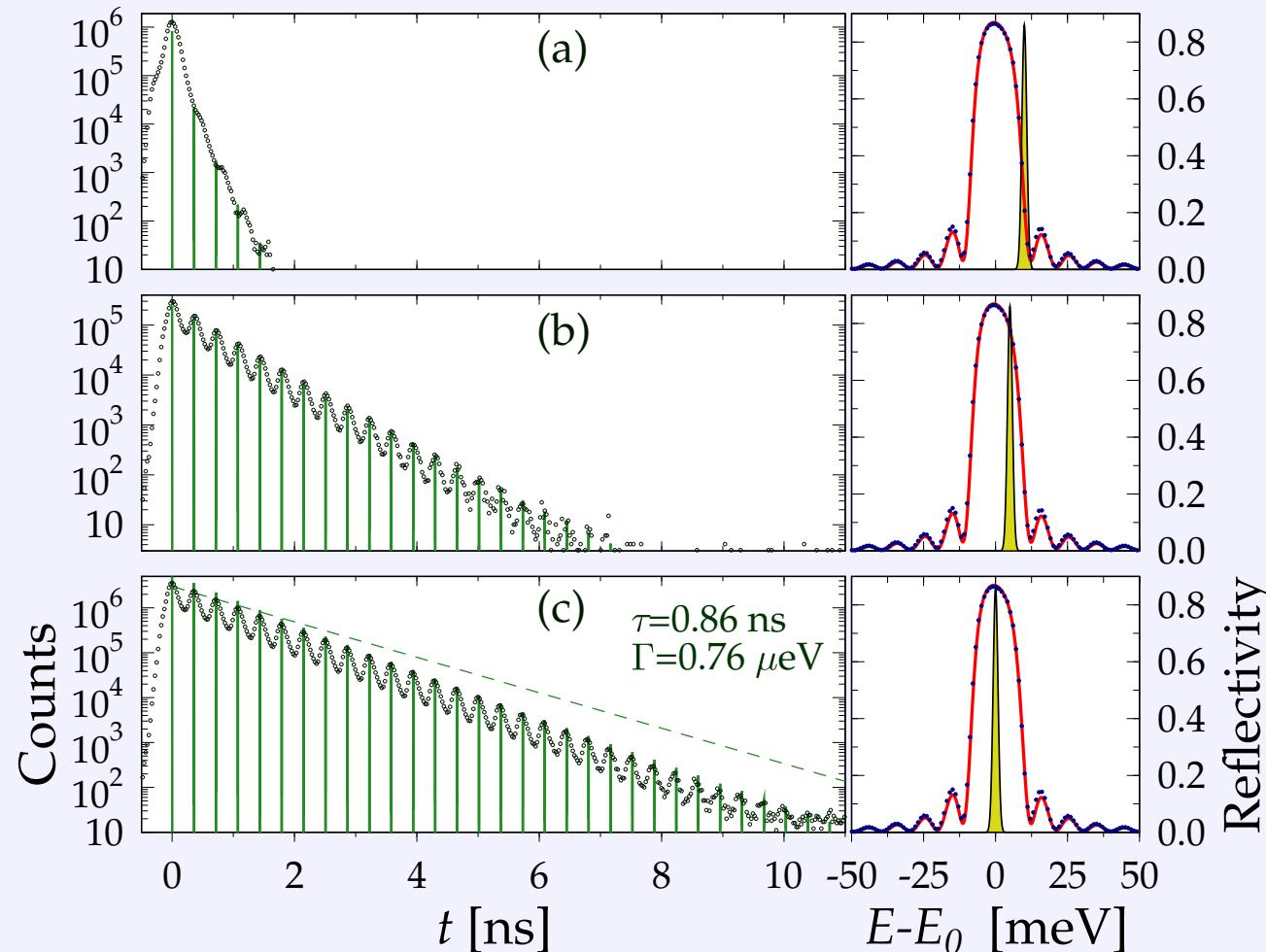
$\text{Al}_2\text{O}_3(0\ 0\ 0\ 30)$, $E_0 = 14.315 \text{ keV}$

$\Delta\theta = 15 \mu\text{rad}$

$$E_f = 12 \mu\text{eV} \implies t_f = 330 \text{ ps}$$

$$\Gamma = 0.73 \mu\text{eV} \implies \tau = 0.9 \text{ ns}$$

Time response of the resonator



Left: Time response of an x-ray resonator with the sapphire mirrors. The blue sharp peaks are the theoretical fits.

Right: Reflectivity of the resonator mirrors (green) and the spectrum of the incident radiation (yellow).

$$\begin{aligned}\tau &\simeq 0.86(1) \text{ ns} \implies \Gamma \simeq 0.76(1) \mu\text{eV}; \\ t_f &= 358(1) \text{ ps} \implies F = 2\pi\tau/t_f \simeq 15\end{aligned}$$

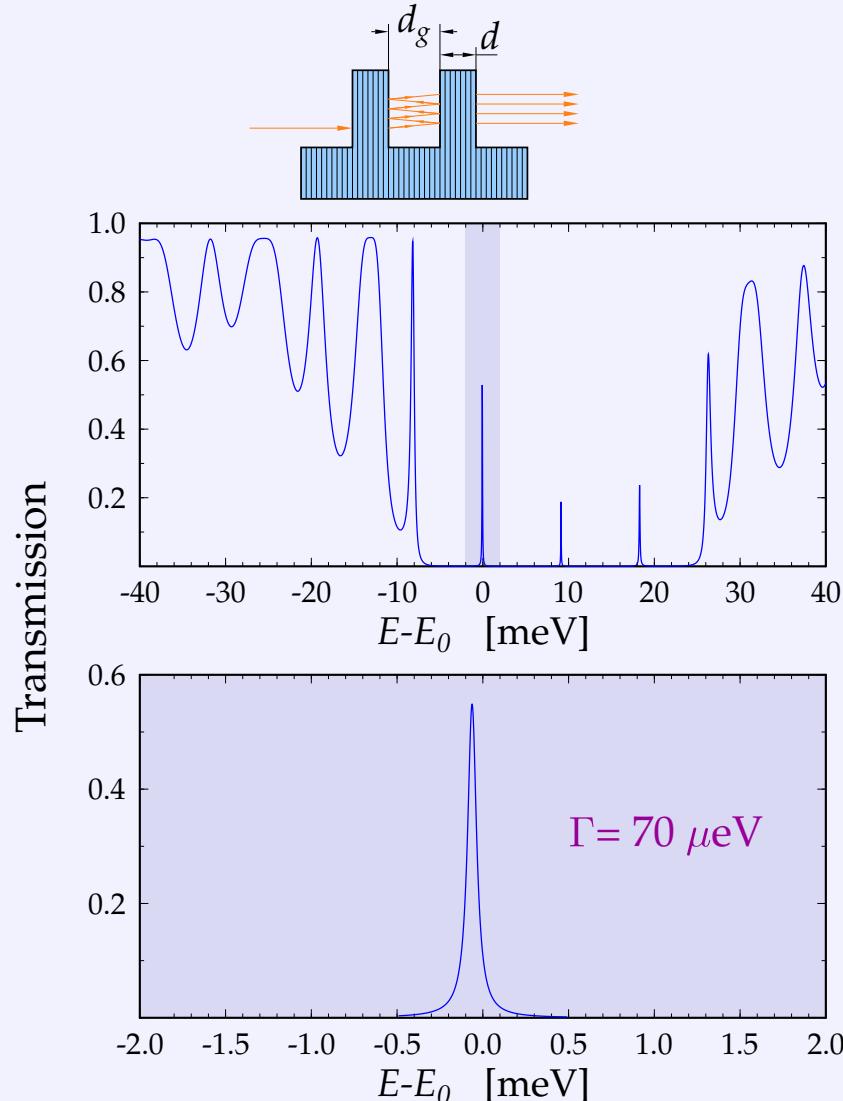
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Potential applications of x-ray Fabry-Pérot interferometers:

- X-ray spectroscopy with μ eV-resolution
- Phase-contrast imaging
- Metrology with Mössbauer radiation
- Delayed lines for ultrafast x-ray science
- X-ray clock
-

Diamond μeV interference filter



X-ray Fabry-Pérot interferometer with diamond x-ray mirrors as μeV interference filter.

$$d \simeq d_g = 60 \mu\text{m}.$$

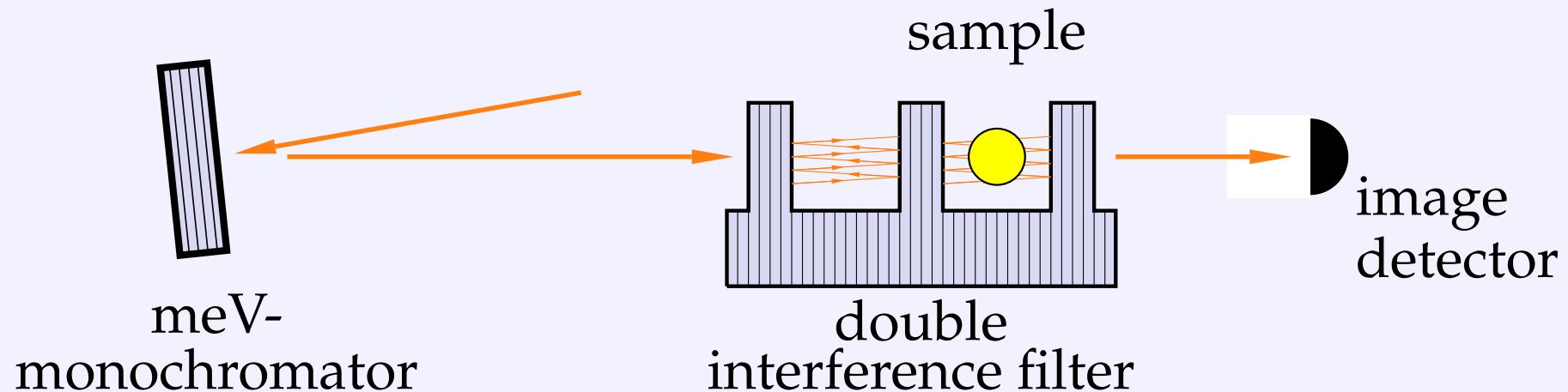
$$\Gamma = 70 \mu\text{eV}$$

$$E_0 = 12 \text{ keV}.$$

$$\Gamma/E_0 \simeq 6 \times 10^{-9}$$

$$\Delta\Theta = 2\sqrt{\Gamma/E_0} \simeq 0.15 \text{ mrad}$$

Phase-contrast imaging



X-ray Fabry-Pérot interferometer possess enhanced phase sensitivity due to multiple-beam interference.

$$\Delta\phi = 2\pi/F.$$

Sensitive to $\simeq 20$ nm layer of carbon.

Factor $F \simeq 100$ smaller radiation dose.

Metrology with Mössbauer radiation

Metrology

Time measurements with ^{133}Cs atomic clock:

Uncertainty: 1.7×10^{-15}

(NIST-F1 cesium fountain atomic clock, 1999)

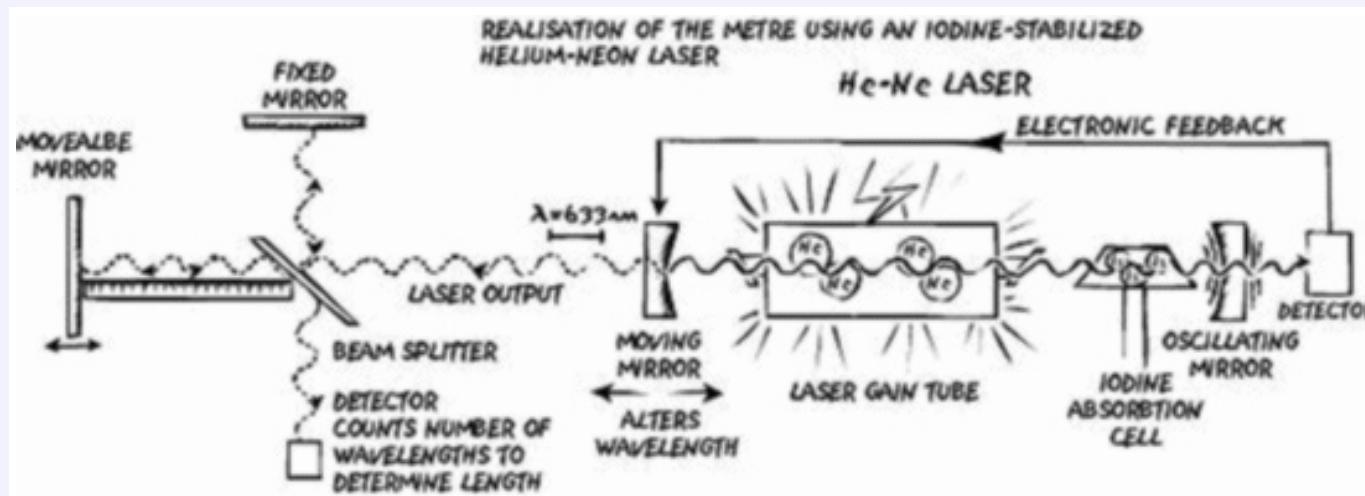
Length = speed of light \times time.

$$c = 299\ 792\ 458\ \text{m s}^{-1} \text{ (exact)}$$

Realization:

λ_s of I-stabilized He-Ne laser:

$$\lambda_s = c/\nu_s \simeq 633\ \text{nm} \text{ with } \Delta\nu_s/\nu_s \simeq 2 \times 10^{-11}.$$



Length standard for nano metrology

1. Lattice constant of Si at 22, 500 °C: $a \simeq 0,543 \text{ nm}$.

Measured in units of λ_s with $\delta a/a \simeq 3 \times 10^{-8}$

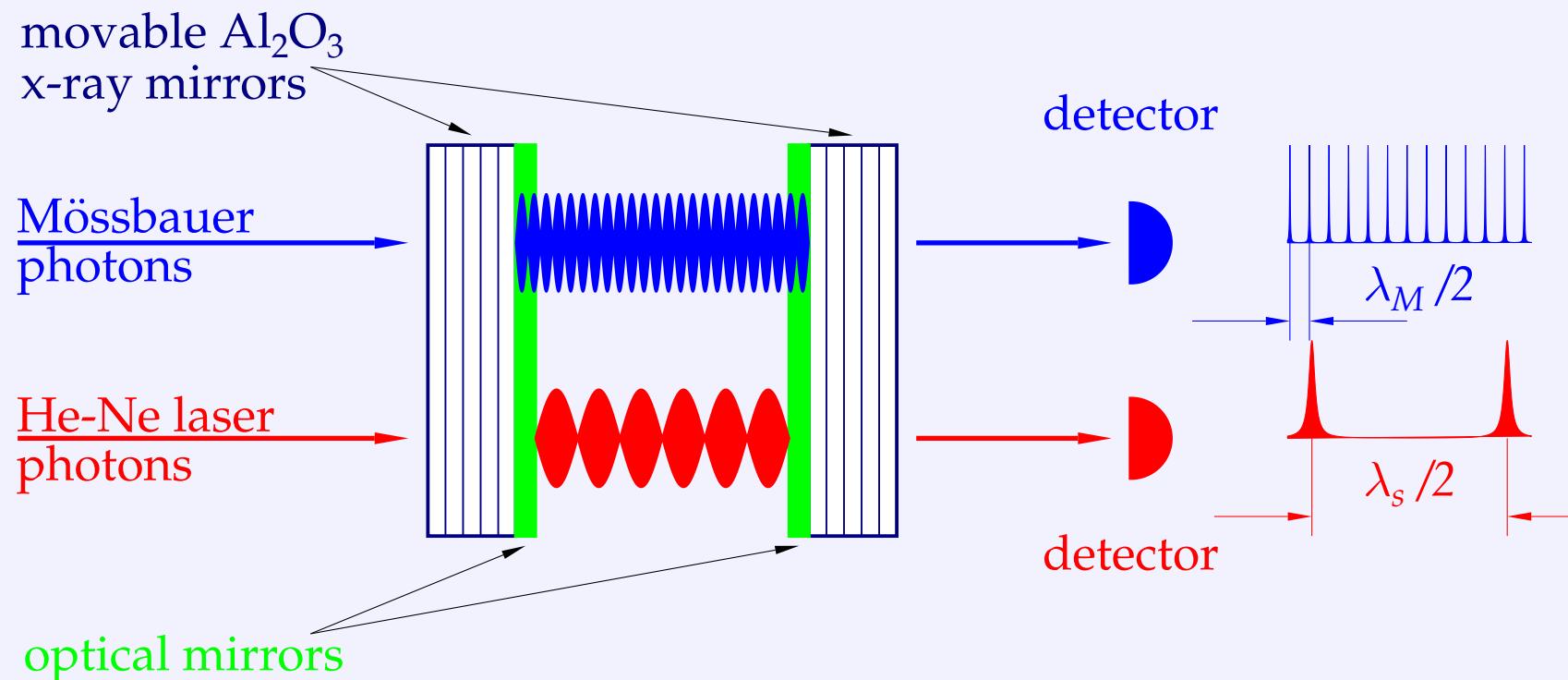
Problems? \Rightarrow Reproducibility !

2. Alternative: The wavelength λ_M of the Mössbauer radiation of ^{57}Fe .

Easy reproducible with $\delta\lambda_M/\lambda_M \simeq 10^{-15}$

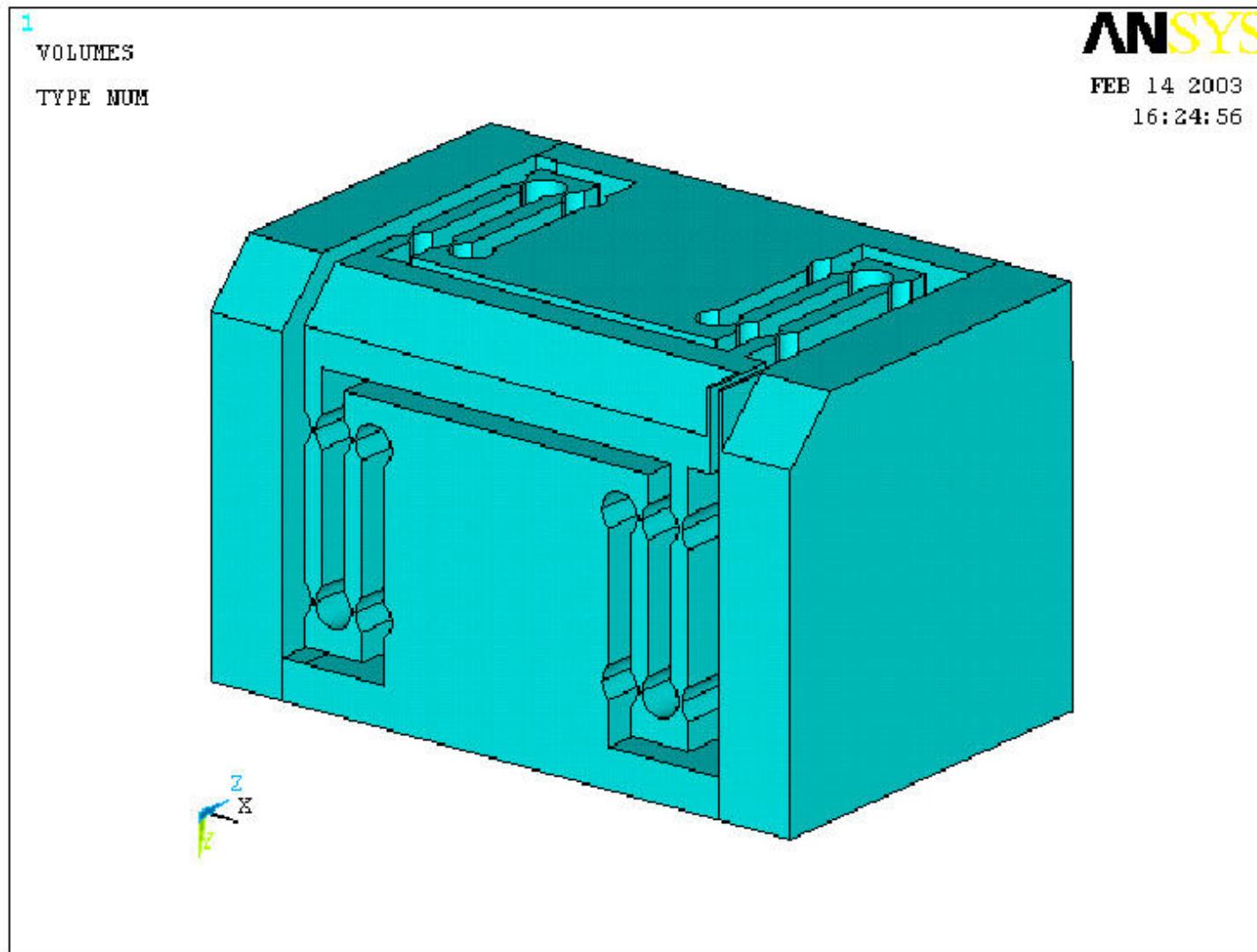
How to measure the absolute value of λ_M ? (1)

Direct linkage of λ_M to λ_s :
measuring the Mössbauer wavelength in terms of the wavelength of the He-Ne laser.
 $\lambda_M / \lambda_s \simeq 1.4 \times 10^{-4} !!!$



Combined Fabry-Pérot Interferometer for He-Ne laser- und Mössbauer radiation.

Sapphire x-ray Fabry-Pérot interferometer with moving mirrors



Thanks to

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